# IA Cliquet Greeks Calculation and Attribution Analysis

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AGL are selling Indexed Annuity (IA) linked to the S&P 500 index(SPX) with Cliquet rider. AGL would like to hedge the equity guarantees embedded in this product with a dynamic hedging program through Market Risk Management group in Woodland Hills. MRM will evaluate the IA block’s reserve and Greeks and rebalance the hedges periodically. The document addresses the calculation of IA Cliquet block’s Greeks and hedge positions.

Background

The one year monthly Cliquet option’s payoff function can be written as formula (1).

(1)

P is premium amount

a is participation rate

St is the index level at the end of each month

S0 is the index level at the beginning of one year term

Cliquet Greeks Calculation

Value:

For the lack of close form solution, the value of a Cliquet option( is calculated based on the Monte Carlo simulation. The volatility models used for the Monte Carlo simulation are either Bates model or Heston model. The simulation time step is daily. After a large enough sample size(we uses 50K paths for the initial implementation) are generated for the equity return paths, the path wise payout of the Cliquet is calculated first and the average of the discounted path wise results is the option value. For inforce Cliquet option, the SPX levels for the past observation dates are known. Only the SPX levels for the future observation dates are taken from the simulation paths.

Delta:

is calculated using the same equity return paths. However, the starting SPX level is bumped up by 0.5 point. All observation on SPX happened in the past should be kept the same. is calculated in the same way.

Gamma:

Vega:

is calculated using the new and

is calculated using the new and

Please note the Vega calculated here is not used for attribution analysis since it is hard to define and under a stochastic volatility model. The attribution analysis on the volatility change is explained in the next section.

Theta:

is the Cliquet value calculated at t+1 business day assuming there is not SPX index level change from t to t+1 day. The original is the Cliquet value calculated at t days. is calculated using a different set of equity return paths. Assume is the closing SPX index level on the valuation day t. In the new set equity return paths, we assume the equity return from t to t+1 is zero. The equity return from t+n to t+n+1 in the new return paths is the same as the return from t+n-1 to t+n in the original set of equity paths. Please note since all the dates used here are business days, we multiple the daily theta by 252 to get to the yearly theta.

The Value, Delta, Gamma, Vega and Theta are additive. The sum of by policy Value, Delta, Gamma, Vega and Theta is the block’s Value, Delta, Gamma and Theta.

Only special treatment is for policies that settle on a nonbusiness day. According to the production spec, the previous business day’s SPX level will be used for the settlement and also be used as initial SPX level in the new term. For that reason, we reset the Cliquet option on the last business day if the actual settlement day falls on a nonbusiness day.

Attribution

Attribution analysis is the tool to explain changes of Cliquet value. The major contributors to the change of Cliquet value are changes in index level, index volatility and time decay. Formula (2) decomposes the changes of the Cliquet value into changes due to Delta, Vega, Gamma, Speed and Theta(time decay). The unexplained change is included as epsilon. Usually the epsilon is a small number.

(2)

is the number of business days between EOP and BOP.

The is calculated using the following method. Revalue the BOP inforce Cliquet policies using the EOP volatility model parameters while all other parameters the same as BOP. The difference between this value and the original BOP value is the change due to vega.

Considering the case of settlement and renewal, the formula (1) becomes (2)

(3)